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F. Casoli, T. Contini, J.M. Hameury, and L. Pagani (eds)

THE ADVECTIVE-ACOUSTIC INSTABILITY IN TYPE II SUPERNOVAE

GALLETTI, P.[1] and FOGLIZZO, T.[1]

Abstract. The puzzle of birth velocities of pulsars (pulsar kicks) could be solved by an asymmetric explosion of type II Supernovae. We propose a simple hydrodynamical mechanism in order to explain this asymmetry, through the advective-acoustic cycle (Foglizzo 2002) : during the phase of stalled shock, an instability based on the cycle between advected perturbations (entropy / vorticity) and acoustic perturbations can develop between the shock and the surface of the nascent neutron star. Eigenfrequencies are computed numerically, improving the calculation of Houck & Chevalier (1992). The linear instability is dominated by a mode $l = 1$, as observed in the numerical simulations of Blondin et al. (2003) and Scheck et al. (2004). The frequency dependence of the growth rate reveals the presence of the advective-acoustic cycle.

1 Stationary accretion above a solid surface

We consider a shocked accretion flow onto a solid surface (where the velocity of the flow is null) at a constant accretion rate. A cooling region is located above the surface and described by the generic function $\mathcal{L} \propto \rho^{\beta-\alpha} P^{\alpha}$ as in Houck & Chevalier (1992), hereafter HC92. The basic equations of the flow are the continuity equation, the Euler equation and the entropy equation. This latter is : $\frac{\partial S}{\partial t} + (\vec{v}.\vec{\nabla})S + \frac{\mathcal{L}}{P} = 0$, where a measure of the entropy is defined by $S \equiv 1/(\gamma-1)\log(P/\rho^{\gamma})$. These equations are perturbed and projected onto spherical harmonics Y_m^l.

The jump conditions at the shock $r_{\rm sh}$ are given by the Rankine-Hugoniot relations for the stationary quantities. The perturbations at the shock are evaluated taking into account the displacement $\Delta\xi$ of the shock position and its velocity $\Delta v = -i\omega\Delta\xi$. In particular, the perturbations of the transverse velocity are as

[1] SAp, CEA-Saclay, Orme des Merisiers 91191 Gif sur Yvette

follows (Landau & Lifchitz 1987) :

$$\delta v_\theta = \frac{v_1 - v_2}{r_{\rm sh}} \frac{\partial \Delta\xi}{\partial\theta} \quad (1.1)$$

$$\delta v_\phi = \frac{v_1 - v_2}{r_{\rm sh}\sin\theta} \frac{\partial \Delta\xi}{\partial\phi} \quad (1.2)$$

where v_1 and v_2 are the pre-shock and post-shock velocities of the flow.
These transverse velocity perturbations at the shock are not null for non-radial perturbations. This contrasts with Eq. (51) of HC92, who did not allow for transverse velocity perturbations at the shock.

The eigenfrequency ω is a complex number (ω_r, ω_i) such that the velocity perturbation satisfies a wall type condition $(\delta v/v = 0)$ at the surface of the accretor. The imaginary part ω_i of the eigenfrequency is the growth rate of the perturbations.

2 Calculations of the eigenmodes

We performed several calculations of the fundamental modes (an example with $\gamma = 5/3$, $\alpha = 1/2$ and $\beta = 2$ is shown in Fig. 1). The mode $l = 1$ is always the most unstable. This result differs from the analysis made by HC92 because of the error in their boundary conditions at the shock.

The advective-acoustic instability is based on the cycle between advected perturbations (entropy / vorticity) and acoustic perturbations between the shock and the surface. A reference timescale τ for this mechanism is equal to the accretion time from the shock to the coupling region near the surface plus the time for an acoustic wave to reach the shock :

$$\tau \equiv \int_{r_*}^{r_{\rm sh}} \frac{1}{1-\mathcal{M}} \frac{dr}{|v|} \quad (2.1)$$

The acoustic time $t_{\rm ac}$ is defined by the time needed for an acoustic wave to propagate from the shock to the accretor and then back up to the shock, $\omega_{\rm ac} \equiv 2\pi/t_{\rm ac}$ being the pulsation associated to this acoustic time :

$$t_{\rm ac} \equiv \int_{r_*}^{r_{\rm sh}} \frac{2}{1-\mathcal{M}^2} \frac{dr}{c} \quad (2.2)$$

On Figs. 1, 2, the growth rate ω_i is at best comparable to $\omega_{\rm sh} \sim 1/\tau$ and the pulsation ω_r of the fundamental unstable modes is close to $2\pi/\tau$, as expected in the advective-acoustic mechanism. The frequency dependence of the growth rate of the eigenmodes shows an oscillatory behaviour with a period comparable to $\omega_{\rm ac}$ (Fig. 2). Such oscillations are expected in the advective-acoustic instability, as a consequence of the modulation of the advective-acoustic cycle by the purely acoustic cycle (Foglizzo 2002). This interpretation is confirmed by measuring the

ratio $\tau/t_{\rm ac} \sim 4.5 - 6.5$ which is comparable to the number of modes found per oscillations (Foglizzo 2002).
We note that for big cavities, the most unstable eigenmodes correspond to low frequencies, in the "pseudo-sound" regime ($\omega_r < \omega_{\rm ac}$). Unstable eigenmodes are also found in the acoustic regime ($\omega_r > \omega_{\rm ac}$), with a smaller growth rate however (Fig. 2).

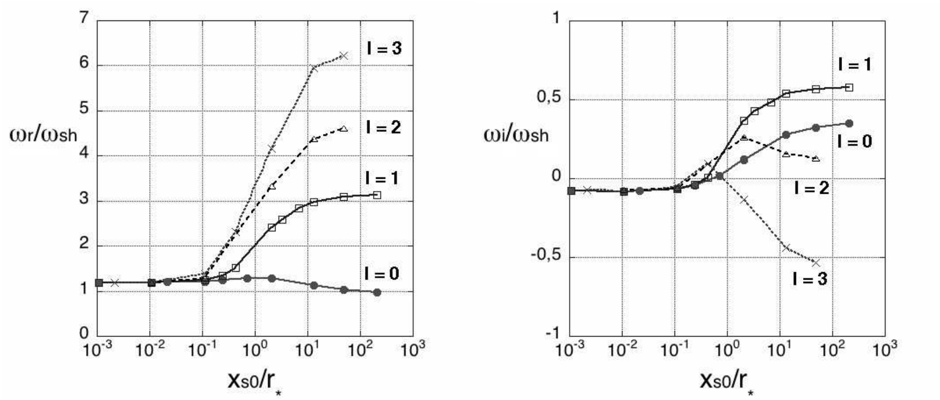

Fig. 1. For $\gamma = 5/3$, $\alpha = 1/2$ and $\beta = 2$, numerical calculations of the frequency ω_r and the growth rate ω_i in units of $\omega_{\rm sh} \equiv -v_{\rm sh}/(r_{\rm sh} - r_*)$ of the fundamental modes $l = 0$, 1, 2, 3 depending on the size of the cavity $x_{s0}/r_* \equiv (r_{\rm sh} - r_*)/r_*$, as in Houck & Chevalier (1992). The degree l of the modes is indicated on each curve.

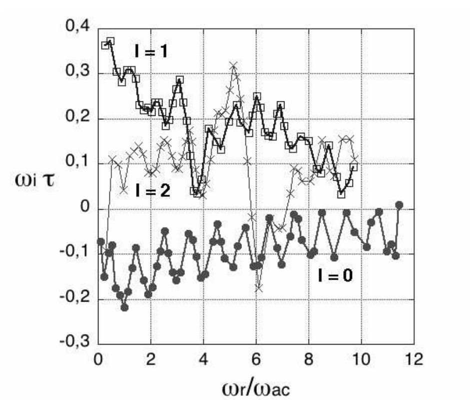

Fig. 2. For $\gamma = 4/3$, $\beta = 2$, $\alpha = 1/2$, and a cavity $r_{\rm sh}/r_* \sim 12$, numerical calculations of the frequency ω_r in units of $\omega_{\rm ac}$ and the growth rate ω_i in units of $\omega_{\rm sh}$ of radial $l = 0$ and non-radial $l = 1, 2$ eigenmodes.

On Fig. 3, for $\gamma = 4/3$ and a cooling function described by $\alpha = 6$, $\beta = 1$, relevant to the phase of a stalled shock in type II Supernovae (Bethe & Wilson 1985), an unstable mode $l = 1$ can also be found for big enough cavities ($r_{\rm sh}/r_* \gtrsim 3.5$). A frequency dependence study, still in progress, shows unstable modes $l = 1$ both in the "pseudo-sound" regime and in the acoustic regime. The radial modes ($l = 0$) are always stable.

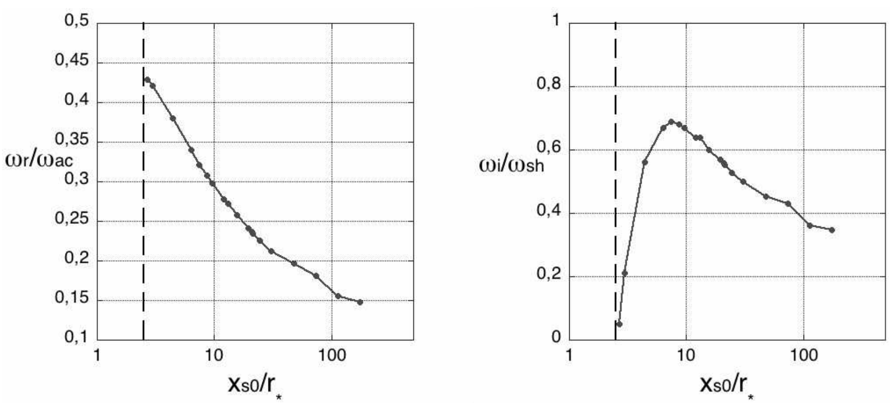

Fig. 3. For $\gamma = 4/3$, $\alpha = 6$ and $\beta = 1$, numerical calculations of the frequency ω_r in units of $\omega_{\rm ac}$ and the growth rate ω_i in units of $\omega_{\rm sh} \equiv -v_{\rm sh}/(r_{\rm sh} - r_*)$, of the first unstable mode $l = 1$, depending on the size of the cavity $x_{s0}/r_* = (r_{\rm sh} - r_*)/r_*$. The vertical dashed line correspond to $r_{\rm sh}/r_* = 3.5$ ($x_{s0}/r_* = 2.5$).

3 Conclusion

The stalled accretion shock above a neutron star is unstable, with a domination of a mode $l = 1$ if the shock radius is large enough. The instability is interpreted as an advective-acoustic cycle modulated by a purely acoustic cycle. The instability is also found when the cooling function mimics neutrino cooling in type II Supernovae. The advective-acoustic cycle is thus a good candidate to seed an asymmetric explosion which could lead to an important birth velocity of the neutron star.

References

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